

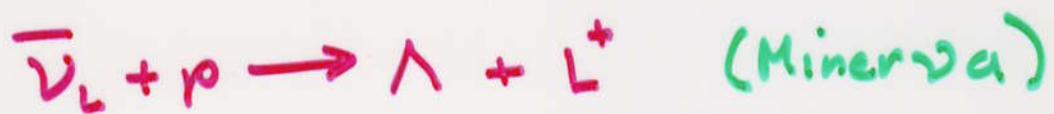
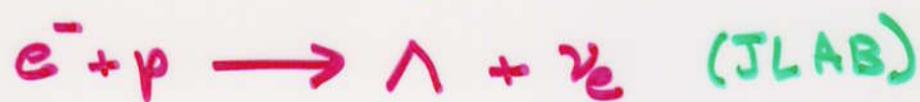
# The Reaction



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Recently there has been some interest in studying semi-leptonic strangeness changing reactions on protons



We can consider the first reaction briefly

### Some Advantages

- (1) Possible to use a reliable beam of electrons which can be varied in energy
- (2) Results can be obtained in terms of only four form factors

(25) There are some disadvantages

- (1) Certain interesting parts of the interaction are not observable due to the small mass of the electron ( $F_p$  and  $F_s$ )
- (2) No asymmetry measurements are possible because the electron is highly relativistic even at threshold for this process

$$E_{e+} = 194.1753 \text{ MeV}$$

Some of the disadvantages of using electrons might be overcome by using more massive leptons (should they become available)



However neutrino beams might be available to look at



where  $L = e, \mu, \tau$

In this talk we concentrate on the neutrino reactions. Although muon anti neutrinos will be available for Minerva we consider  $\bar{\nu}_e$  and  $\bar{\nu}_\mu$  reactions as well.

We will avoid detailed models as much as possible and make use of SU(3) and measured results where possible

The production of  $\Lambda$  hyperons by antineutrinos is well described by the matrix element

$$\begin{aligned} \langle L^+ \Lambda | H_w | \bar{\nu}_e p \rangle &= \frac{G}{\sqrt{2}} \sin \theta_C \bar{\nu}_e \gamma^\mu (1 - \gamma_5) \\ &\times \langle \Lambda | J_\mu^+(o) p \rangle \end{aligned}$$

where the weak current is given by

$$J_\mu(o) = V_\mu(o) - A_\mu(o)$$

All of the unknown quantities are contained in the hadronic matrix element

$$\langle \Lambda | J_\mu^+(o) | p \rangle$$

The case would be the same for  $\Sigma^0$  production.

The vector and axial vector current matrix elements may be written as

$$\langle \Lambda | V_\mu^+(o) | p \rangle = \bar{u}_g [ \gamma_\mu F_V(q^2)$$

$$+ i \frac{F_\mu(q^2)}{2m_p} \sigma_{\mu\nu} g^\nu$$

$$- F_g(q^2) \frac{g_\mu}{2m_p} ] u_i$$

and

$$\langle \Lambda | A_\mu^+(o) | p \rangle = \bar{u}_i [ \gamma_\mu Y_S F_A(q^2) + g_\mu Y_S \frac{F_P(q^2)}{m_\pi}$$

$$+ i \frac{\bar{F}_E(q^2) \sigma_{\mu\nu} g^\nu \gamma_5}{2m_p} ] u_i$$

So we need the six form factors

$F_V, F_\mu, F_g, F_A, F_P, F_E$  but  $F_P$  and  $F_g$  are suppressed in the electron case

$\bar{F}_E$  and  $\bar{F}_S$  would be second class current form factors in an  $n \leftrightarrow p$  transition but not here

From well known SU(3) relationships

$$F_r = -\frac{1}{16} (3 \tilde{F}_r + \tilde{D}_r) \quad r = V, M, A, E, 1, 2, 3$$

which lead to

$$F_V(g^2) = F_V(0) / (1 - g^2/M_V^2)^2$$

$$F_V(0) = 1.2247 \quad M_V = .98 \text{ GeV}/c^2$$

and

$$F_M(g^2) = F_M(0) / (1 - g^2/M_M^2)^2$$

$$F_M(0) = 1.713 / 2m_p \quad M_M = .71 \text{ GeV}/c^2$$

From  $\Lambda$  beta decay

$$F_\Lambda(g^2) = F_\Lambda(0) / (1 - g^2/M_\Lambda^2)^2$$

$$F_\Lambda(0) = .8793 \quad M_\Lambda = 1.25 \text{ GeV}/c^2$$

From a theoretical reference

$$F_E(0) \approx .705 / 2m_p$$

and from other arguments  
the  $q^2$  dependence should be  
similar to  $F_M$

$$F_E(q^2) \approx F_E(0) / (1 - q^2/M_n^2)^z$$

From a similar estimate

We may write

$$F_S(g^2) = F_S(0) / \left(1 - e^2/m_e^2\right)^2$$

$$F_S(0) \approx .344 F_E(0)$$

For  $F_P$  we use a Nambu type relationship

$$F_P = - \frac{F_A m_n (m_i + m_b)}{(g^2 - m_b^2)}$$

Making use of these and the previous form factors we can obtain a matrix element where the lepton mass is not suppressed and calculate the differential cross section

$$\begin{aligned}
|M^2| = & \frac{1}{m_\tau m_\nu} \left[ \frac{4 |F_V|^2}{M_f M_i} [\tau \cdot p_f \nu \cdot p_i + \tau \cdot p_i \nu \cdot p_f - \tau \cdot \nu M_f \cdot M_i] \right. \\
& + \frac{4 |F_A|^2}{M_f M_i} [\tau \cdot p_f \nu \cdot p_i + \tau \cdot p_i \nu \cdot p_f + \tau \cdot \nu M_f \cdot M_i] \\
& + \frac{2 |F_P|^2}{M_f M_i m_\pi^2} [m_\tau^2 \tau \cdot \nu (\nu \cdot p_i - \tau \cdot p_i - M_i (M_f - M_i))] \\
& + \frac{|F_S|^2}{2 M_f M_i m_p^2} [m_\tau^2 \tau \cdot \nu (\nu \cdot p_i - \tau \cdot p_i + M_i (M_f + M_i))] \\
& - \frac{8 F_V F_A}{M_f M_i} [\tau \cdot \nu (\tau \cdot p_i + \nu \cdot p_i) - m_\tau^2 \nu \cdot p_i] \\
& + \frac{F_M F_E}{M_f M_i m_p^2} [(\tau \cdot \nu + \tau \cdot p_i - m_\tau^2) \nu \cdot p_i) m_\tau^2] \\
& - \frac{2 F_V F_S}{M_f M_i m_p} [m_\tau^2 \nu \cdot p_i (-M_i - M_f) + \tau \cdot \nu M_i) \\
& - \frac{4 F_A F_P}{M_f M_i m_\pi} [m_\tau^2 (\nu \cdot p_i (M_f - M_i) + \tau \cdot \nu M_i)] \\
& - \frac{4 F_A F_M}{M_f M_i m_p} [\tau \cdot \nu (\tau \cdot p_f + \nu \cdot p_f) M_i + \tau \cdot \nu (\tau \cdot p_i + \nu \cdot p_i) M_f \\
& - m_\tau^2 (\nu \cdot p_f M_i + \nu \cdot p_i M_f)] \\
& + \frac{4 F_V F_M}{M_f M_i m_p} [\tau \cdot \nu (\nu \cdot p_i - \tau \cdot p_i) (M_f - M_i) + 2 (\tau \cdot \nu)^2 M_i \\
& - m_\tau^2 (\frac{1}{2} \nu \cdot p_i (M_f - M_i) + \frac{3}{2} \tau \cdot \nu M_i)] \\
& + \frac{|F_M|^2}{M_f M_i m_p^2} [2 \tau \cdot \nu (\tau \cdot p_i \tau \cdot p_f + \nu \cdot p_i \nu \cdot p_f + \tau \cdot \nu M_f M_i) \\
& (-2 \nu \cdot p_i \nu \cdot p_f - \frac{1}{2} \tau \cdot \nu p_i \cdot p_f - \frac{3}{2} \tau \cdot \nu M_f M_i) m_\tau^2] \\
& + \frac{4 F_E F_V}{M_f M_i m_p} [M_i (\tau \cdot \nu (\tau \cdot p_f + \nu \cdot p_f) - m_\tau^2 \nu \cdot p_f) \\
& - M_f (\tau \cdot \nu (\tau \cdot p_i + \nu \cdot p_i) - m_\tau^2 \nu \cdot p_i)] \\
& + \frac{|F_E|^2}{M_f M_i m_p^2} [2 \tau \cdot \nu (\tau \cdot p_i \tau \cdot p_f + \nu \cdot p_i \nu \cdot p_f - \tau \cdot \nu M_f M_i) \\
& (-2 \nu \cdot p_i \nu \cdot p_f - \frac{1}{2} \tau \cdot \nu p_i \cdot p_f + \frac{3}{2} \tau \cdot \nu M_f M_i) m_\tau^2] \\
& + \frac{4 F_A F_E}{M_f M_i m_p} [M_i (\tau \cdot \nu (\tau \cdot p_f - \nu \cdot p_f) + \frac{1}{2} \nu \cdot p_f m_\tau^2) \\
& \left. M_f (\tau \cdot \nu (\tau \cdot p_i - \nu \cdot p_i) + \frac{1}{2} \nu \cdot p_i m_\tau^2) \right]
\end{aligned}$$

The differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{m_\nu m_L m_S}{16\pi^2 E_\nu} \frac{|M|^2}{|M_i + E_\nu - \frac{E_L E_\nu}{|G|} \cos\theta|}$$

- (1)  $|M|^2$  is proportional to at least one power of  $E_\nu$  and either  $E_L$  or  $|G|$
- (2) The form factors are largest when  $g^2$  is smallest
- (3) the  $E_\nu$  in the denominator cancels much of the direct  $E_\nu$  dependence

So  $|M|^2$  is large when  $E_L$  and the form factors are large but that is when  $g^2$  is small.

We refer to terms proportional to  
 $|F_p|^2$  etc as direct terms  
and  $F_A F_V$  etc as interference  
terms

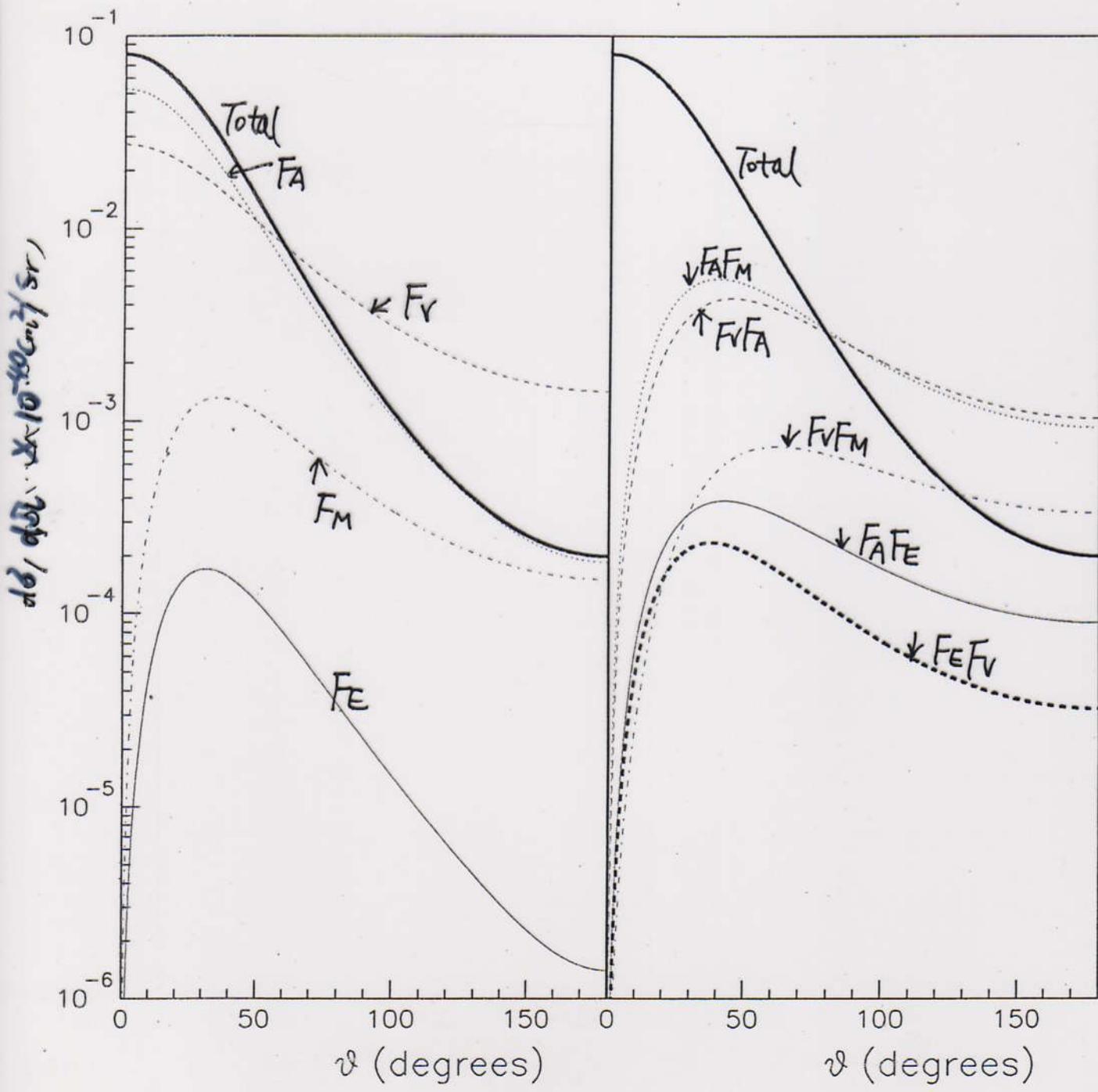
We note that  $F_A F_V$ ,  $F_A F_M$ ,  $F_B F_V$   
and  $F_A F_p$  are negative over all or  
part of their ranges. The absolute  
values are given here

This is why some of the direct  
terms are larger than the cross  
sections over some of the range of  
energy.

So we look at some results

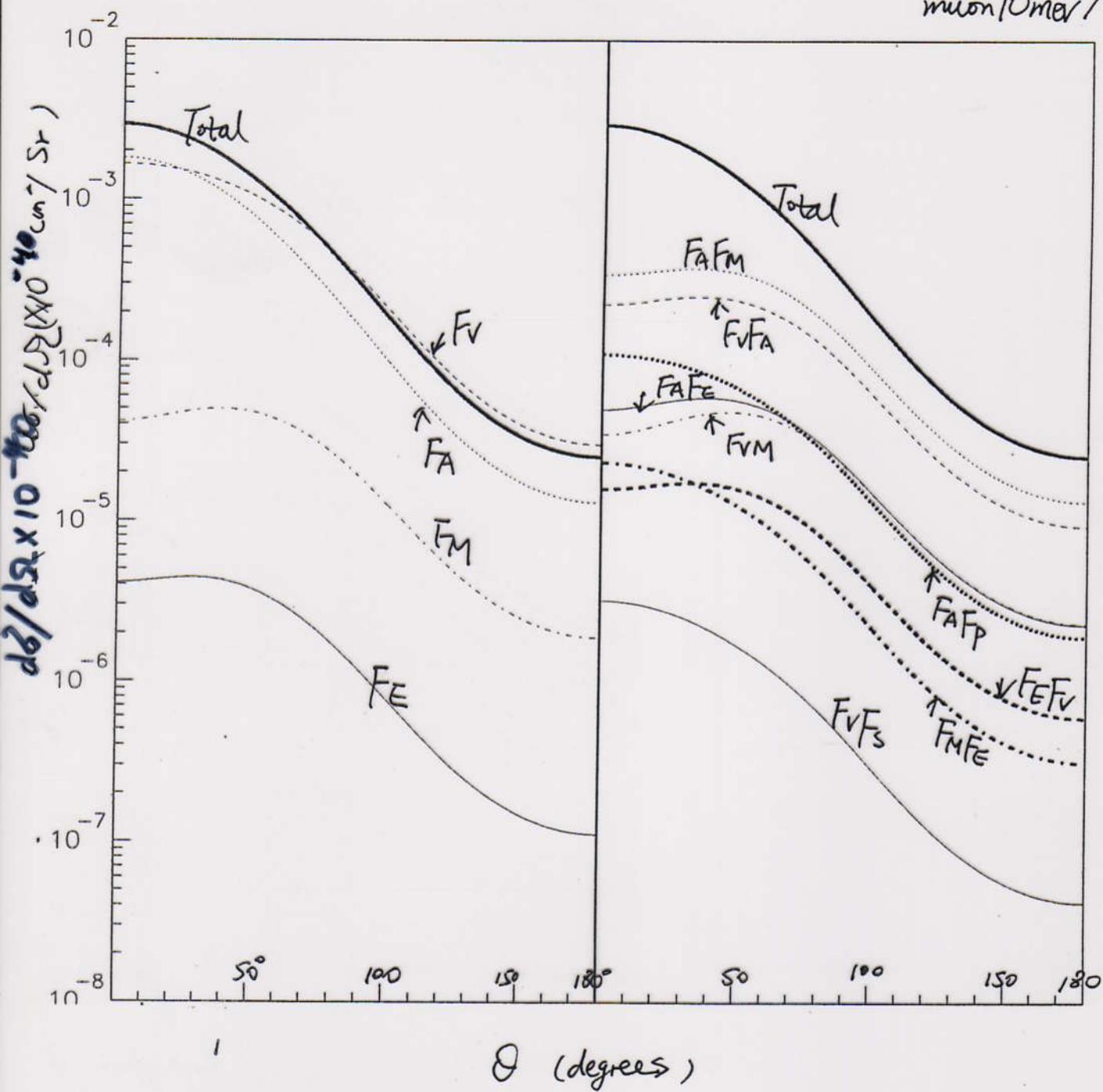
$\Lambda$  production

electron 500mev



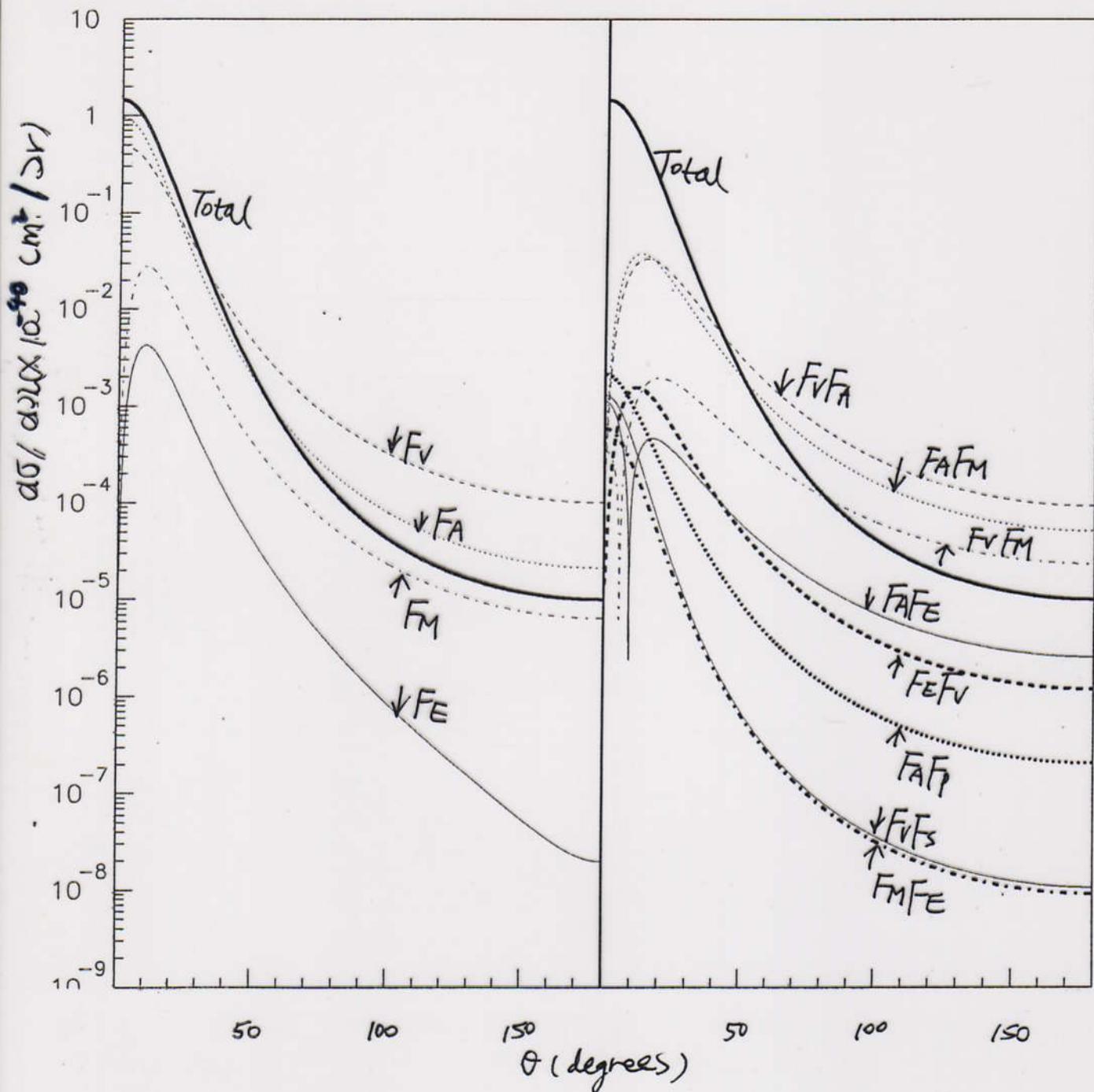
$\Lambda$  production

muon/Omega  $\Lambda$



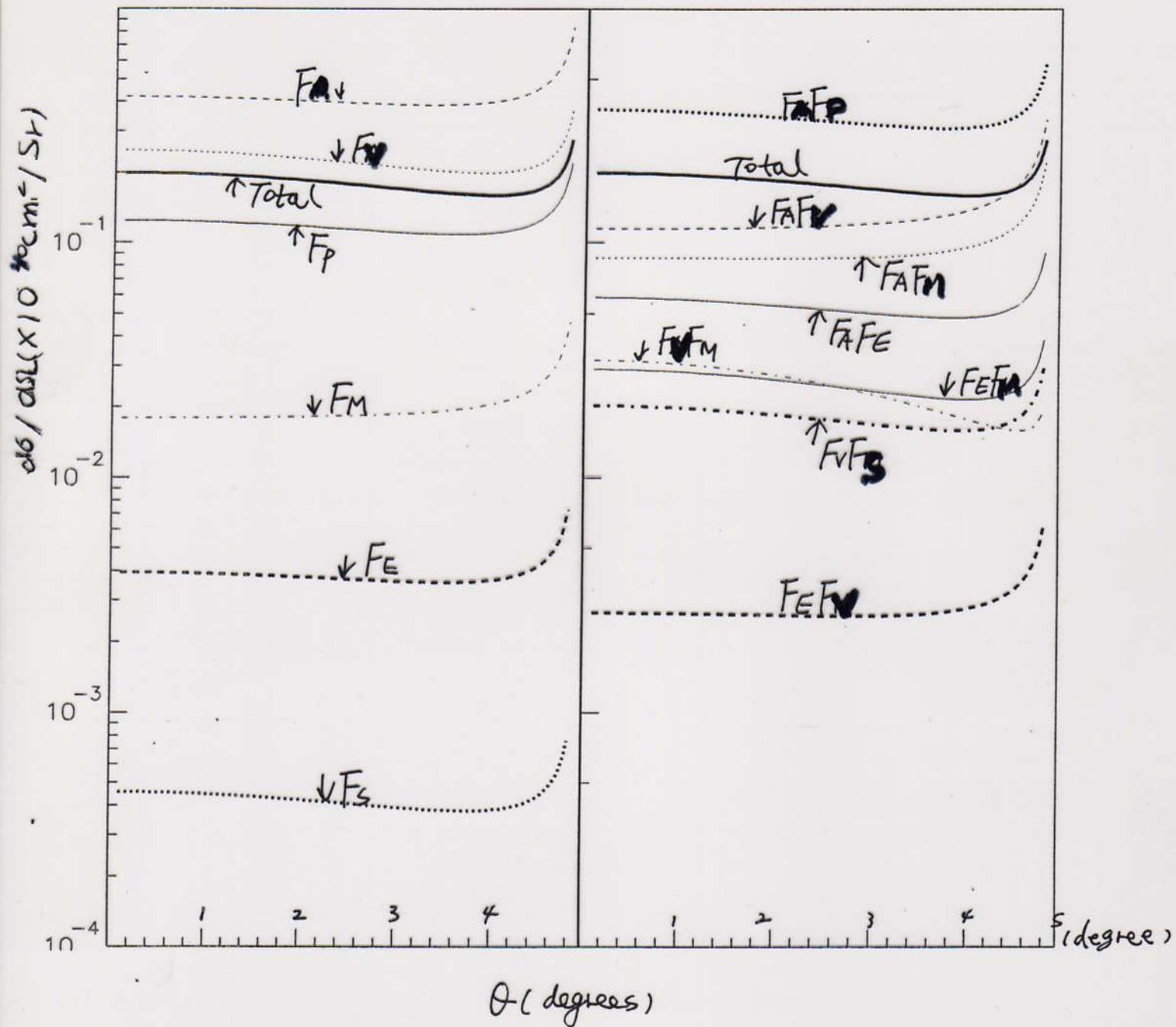
$\wedge$  production

muon 2000 meV

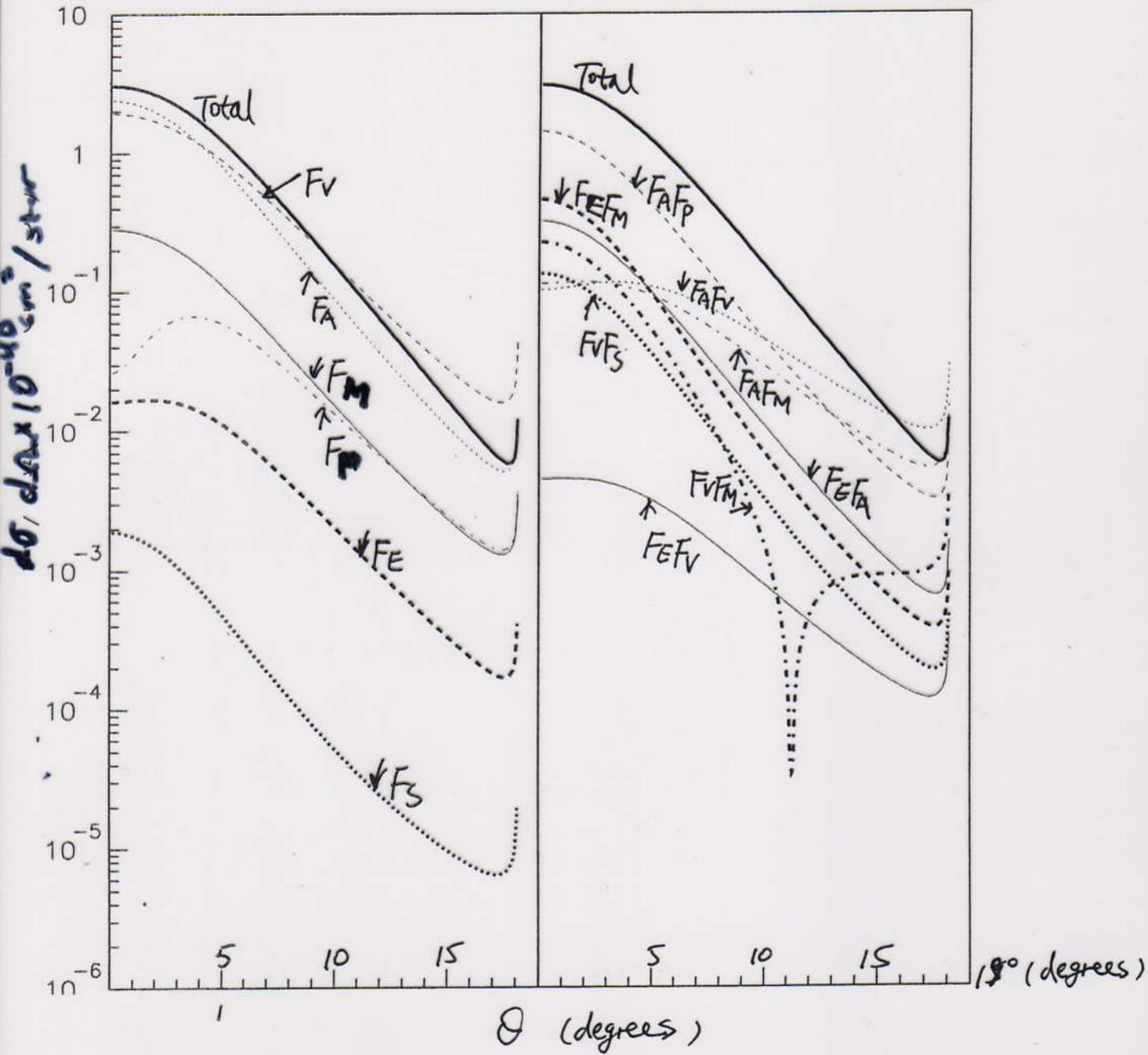


$\Lambda$  production

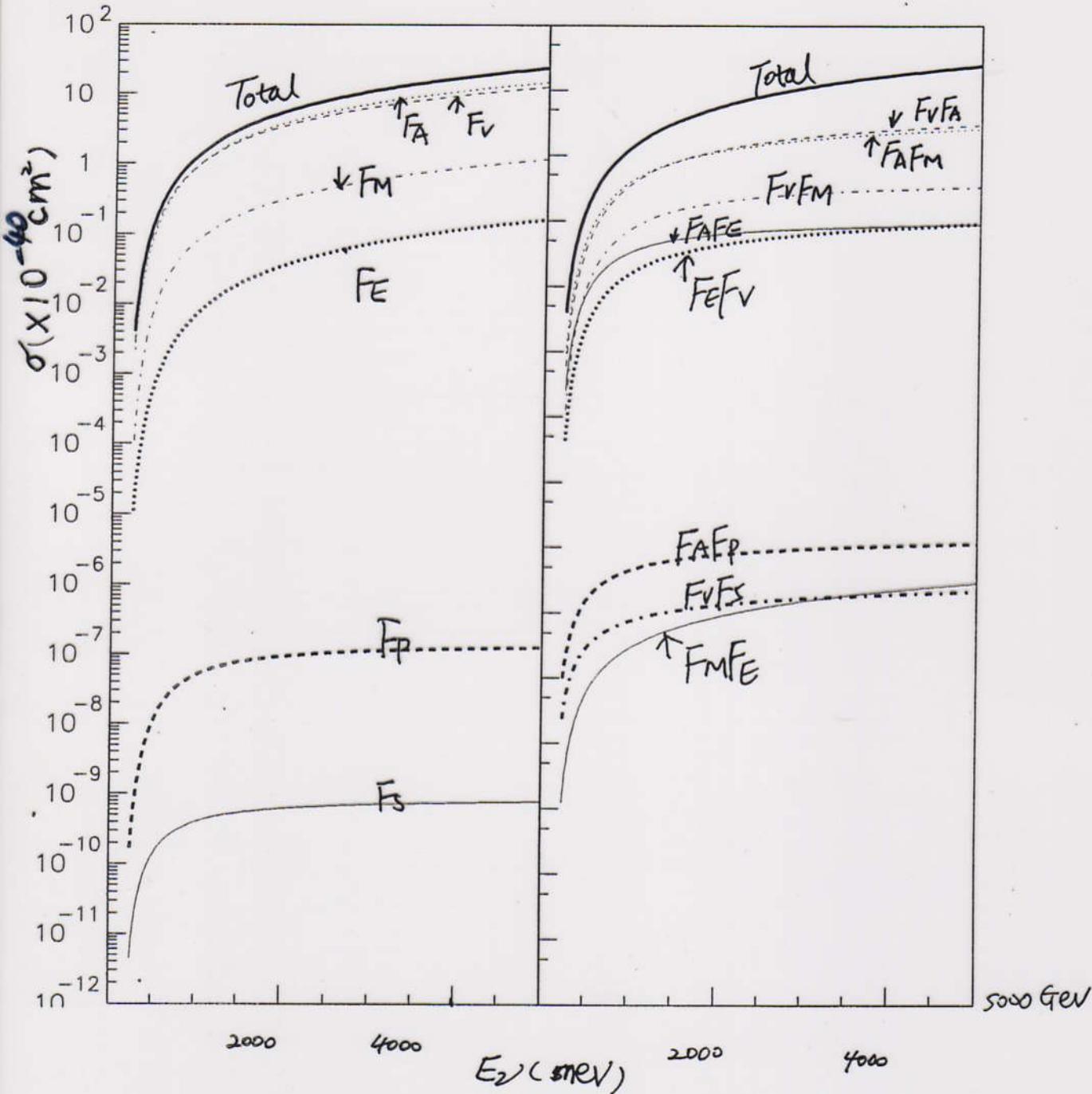
Tall 100 mev



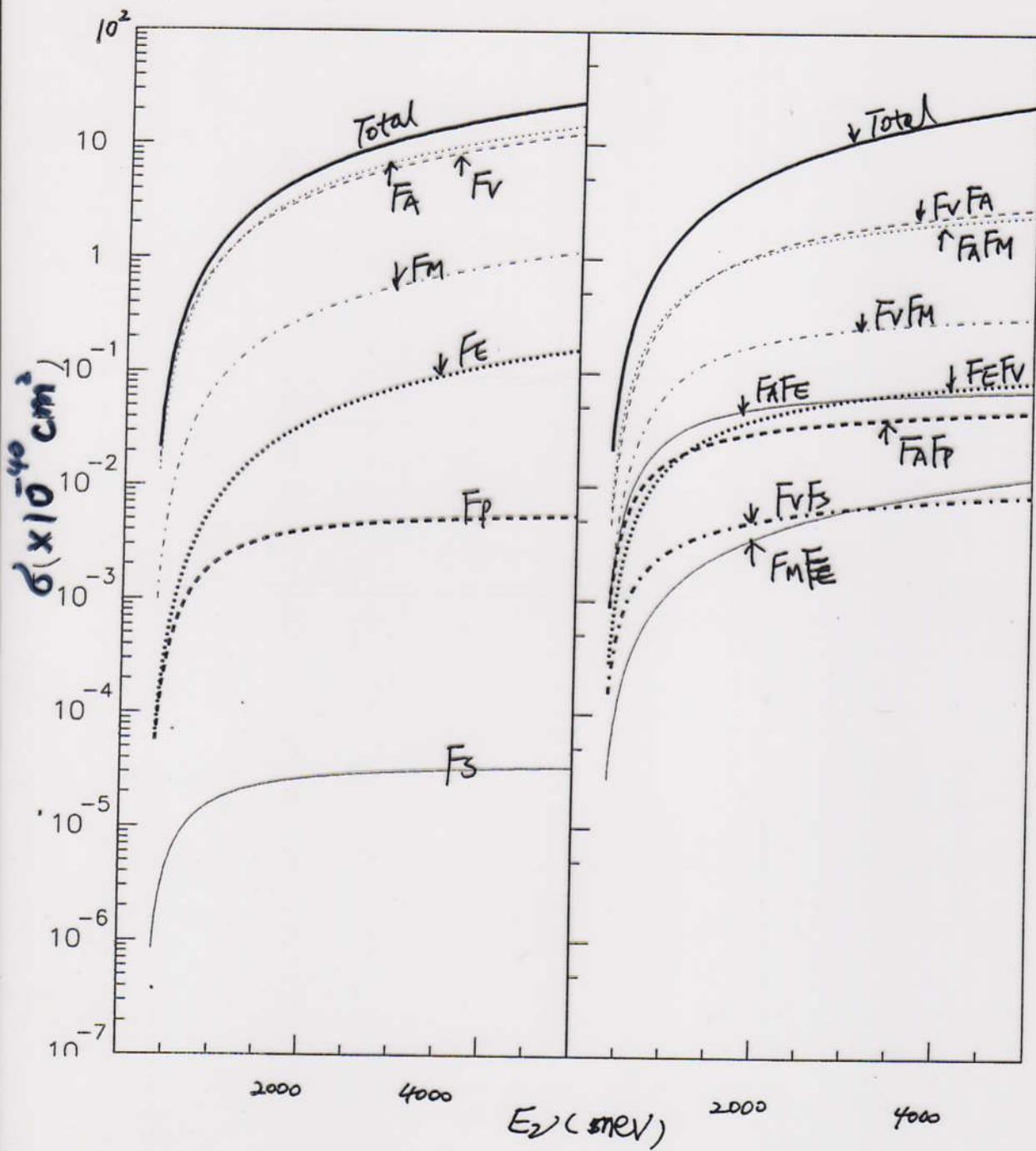
Tau 2020 1



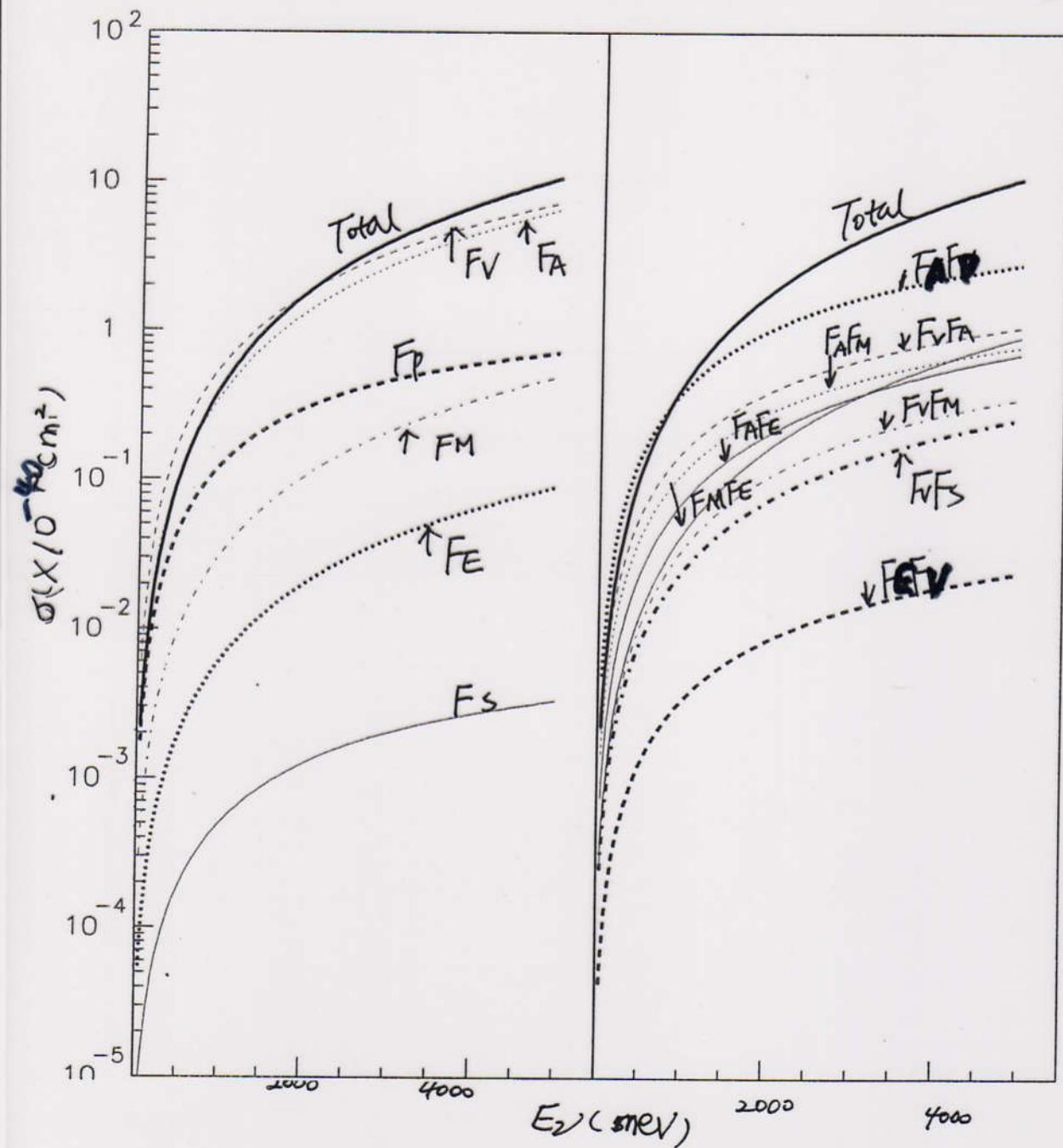
Total electron  $\wedge$



Total muon  $\Lambda$



$\Lambda$  production  
Total Tail



So what might be learned

1. If  $e^- + p \rightarrow \Lambda + \bar{\nu}_e$  is run at JLAB,  $F_V, F_A$  and possibly  $F_M$  might be determined
2. If differential cross sections were available for the electron or muon neutrinos, then  $F_A(q^2)$ ,  $F_V(q^2)$  and  $F_M(q^2)$  might possibly be extracted. This would be a very useful test of the SU(3) form factors, which Cabibbo has speculated might work well even at large  $q^2$
3. If only total cross sections are available for these reactions, then would serve as a consistency test for JLAB or calculated form factors
4. Should  $\bar{\nu}_e$  beams ever become available probably  $F_P$  could be determined.
5. Probably  $F_S$  and  $F_B$  cannot be determined

Another possibility



This might also be run at MINERVA

The matrix elements have a similar form with

$$F_V(q^2) = F_V(0) / (1 - q^2/M_V^2)^2$$

$$F_V = -0.707 \quad M_V = 0.98 \text{ GeV}/c^2$$

$$F_M(q^2) = F_M(0) / (1 - q^2/M_M^2)^2$$

$$F_M(0) = -0.437 / 2m_p \quad M_M = 0.76 \frac{\text{GeV}}{c^2}$$

$\Sigma^+$  does not decay weakly but  
 $\Sigma^-$  does

$$\text{From } [I^+, J_\pi] = 0$$

$$\langle p | [I^+, J_\pi] | \Sigma^- \rangle = 0$$

$$\frac{1}{\sqrt{2}} \langle n | J_\pi | \Sigma^- \rangle = \langle \nu | J_\pi | \Sigma^+ \rangle$$

Thus

$$\frac{F_A(\xi^0)}{F_V(\xi^0)} = \frac{F_A(\xi^*)}{F_V(\xi^*)} = -0.34 \pm 0.017$$

$$F_A(\xi^*) = F_A(0) / (1 - \xi^2/M_A^2)^2$$

$$F_A(0) = 0.24, M_A = 1.25 \text{ GeV}/c^2$$

As before we use an estimate for  $F_E(0) \approx .705/2m_p$

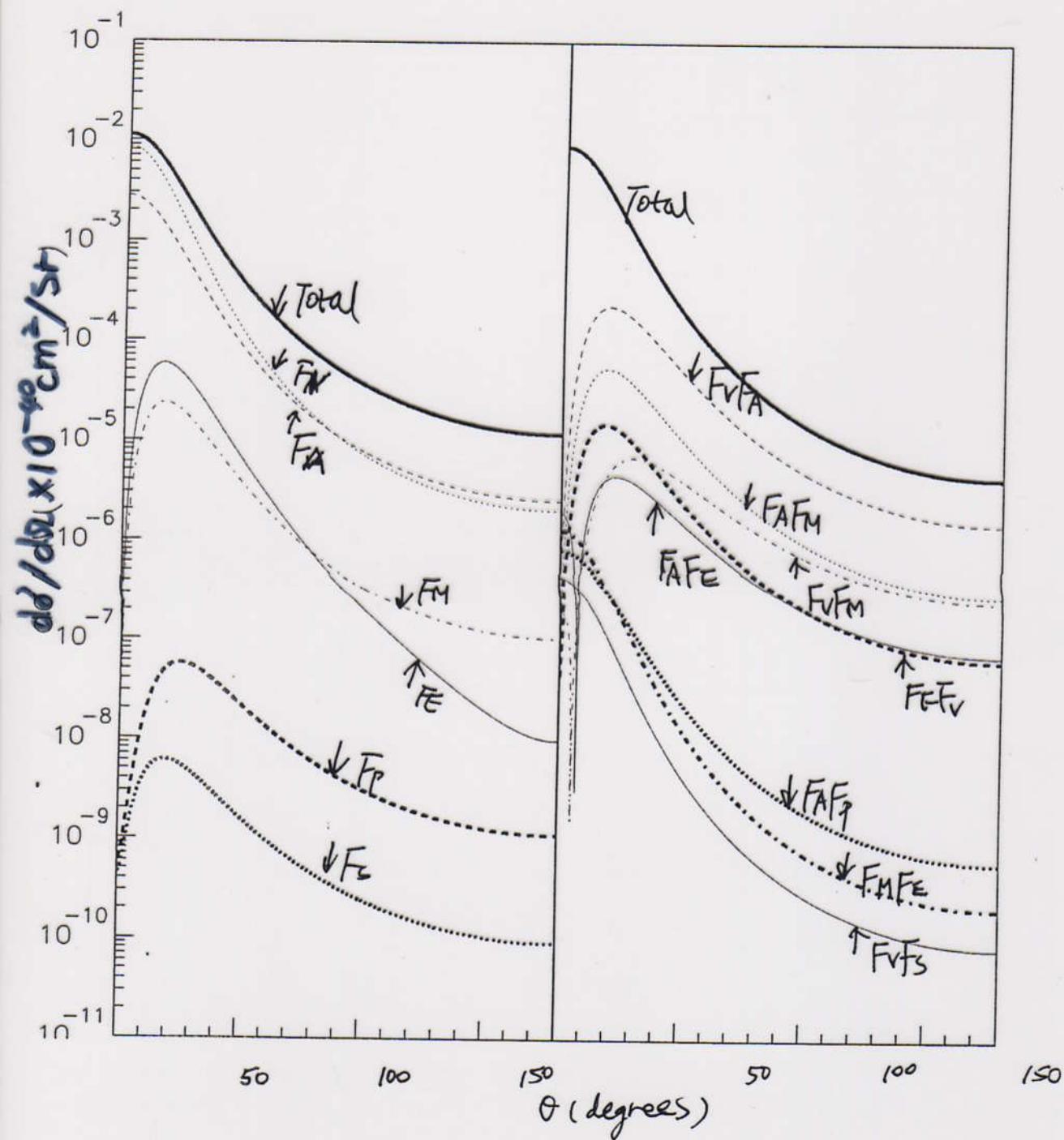
$$F_E(\xi^*) = F_E(0) / (1 - \xi^2/M_E^2)^2$$

$$F_S(0) = .344 F_E(0)$$

The matrix elements are otherwise the same and so we obtain results

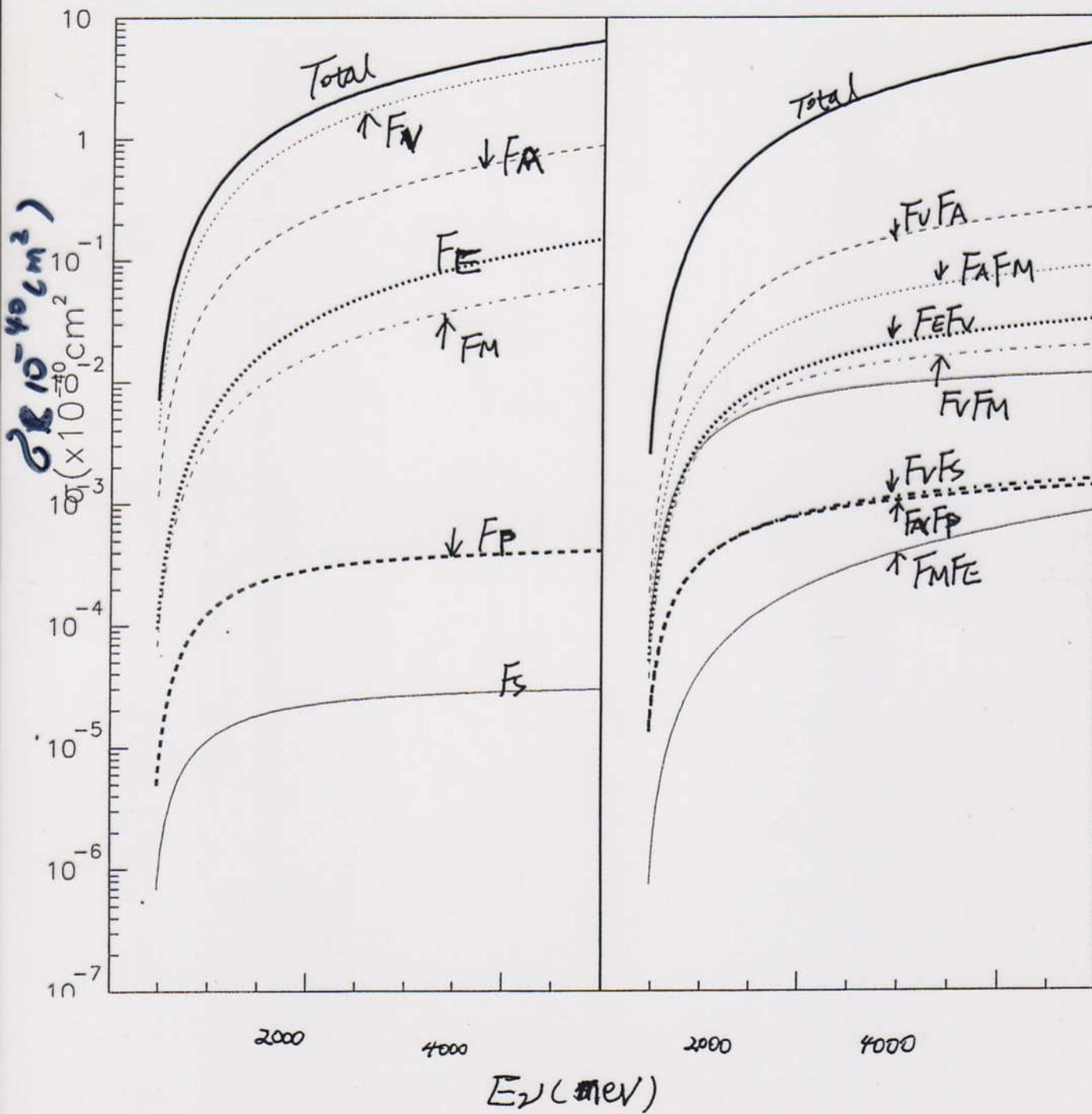
$\Sigma^0$  production

muon 2000 meV



$\Sigma^0$  production

Total muon



so for a cross section a little less than half of that for the  $\Lambda$  case, there is domination by the vector current form factor  $F_V$ , thus this reaction might be a good test of the SU(3) form factors.